The next few pages are more practice problems for you to use as review for Exam 1.

Thank you to Prof Wagaman for sharing these problems.

1. A manufacturing company has come under fire lately for the reliability of its products. Assume for a particular type of office chair, the chairs coming off the assembly line have been classified as perfectly fine, defective, or seriously defective. Furthermore, assume there are 3 different assembly line managers responsible for maintaining the production over the three shifts at the factory. Let X and Y be random variables defined as follows with distribution specified in the table below:

X = 1, perfectly fine chair (PF)
2, defective chair (D)
3, seriously defective chair (SD)

Y = 1, morning shift manager (M)
2, afternoon shift manager (A)
3, overnight shift manager (E)

X\Y	1 (M)	2 (A)	3 (E)
1 (PF)	.15	.42	.23
2 (D)	.04	.05	.06
3 (SD)	.01	.03	.01

a. What proportion of chairs produced by the company are perfectly fine?

b. What proportion of chairs produced by the company are made under the supervision of the afternoon shift manager?

c. Are perfectly fine status (event PF) and overnight shift status (event E) independent? Justify your answer.

d. A company executive sees the table and decides that the afternoon shift manager should be fired because .05+.03 = .08 is larger than .04+.01=.05 and .06+.01=.07. Using probability, provide a rational explanation for why the afternoon shift manager should NOT be fired (and why the company might consider finding another executive).

2. Suppose X is a general discrete random variable taking on the values x=1, 2, 3, 4 and 5, with P(X=x)=cx.

a. What value of c makes this a valid pmf? (Show work to justify your answer.)

b. What are the mean and variance of X?

3. Suppose that you know X, Y, and Z are random variables with the following properties:

E(X)=1	V(X)=1	Cov(X,Y)=-0.4
E(Y)=2	V(Y)=2	Cov(X,Z)=0.5
E(Z)=-1	V(Z)=5	Cov(Y,Z)=2

Compute:

a. E(3X+2Y-Z)

b. V(X+Z)

c. Corr(Y,Z)

4. An insurance policy pays 100 per day for up to 3 days of hospitalization and 50 per day for each day of hospitalization thereafter. The number of days of hospitalization, X, has the pmf

$P(X=k)=(6-k)/15$ for k=1,2,3,4,5, and is 0, otherwise. Determine the expected payout for hospitalization under this policy.

5. A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5. What is the expected amount paid to the company under this policy during a one-year period?

6. A large local club (200 members) has decided to streamline leadership by creating a steering committee of 16 representatives. Note that positions on the steering committee are indistinguishable. (You ***do not*** need to compute the values out for the parts ***a-c*** below.)

a. How many different possible steering committees are there?

b. Of the 200 members, 50 would like being on the committee (like), 100 others would manage alright but aren't overly enthused about the job (manage), and the remainder would hate the job (hate). Assuming the steering committee is randomly selected from among the club members, what is the probability that no one on the steering committee hates the job?

c. If the club wants to guarantee that no one on the steering committee hates the job, and that at least 4 committee members would like the job, how many different possible steering committees are there?

A census (every club member responds) reveals the following attitudes about a new change to the club by-laws, summarized according to steering committee preference. Assume one club member is randomly selected from the club.

	Favor Change	Do Not Favor Change	No Opinion
Would Like	30	15	5
Manage	45	35	20
Hate	15	25	10

d. What is the probability the club member has "manage" preference if the club member is known to not favor the change to the by-laws?

e. Is steering committee preference independent of opinion on changing the by-laws? Provide some probabilistic justification for your answer.

7. Warner's Randomized Response Model is a technique to obtain answers to sensitive survey questions while maintaining confidentiality of results. The survey questions must be yes/no questions. The version proposed here is based on the original 1965 version, though other modifications exist.

Survey participants are asked to flip a fair coin twice and record the flip results for themselves. Then, they are told that if the first coin flip was heads, their answer to the survey question is yes if the second coin flip was heads and no if the second coin flip was tails. If the first coin flip was tails however, they should answer the survey question truthfully: yes or no.

This method was applied for the survey question: "Have you ever cheated as an undergraduate or graduate student?" for 300 business school students. Suppose there are 190 "yes" responses. What is your best estimate as to the proportion of business school students who have ever cheated as an undergraduate or graduate student based on this sample? Show some work to justify your response.

Solutions to further Exam 1 review problems

1. (a) 0.8
   (b) 0.5
   (c) Not independent
   (d) Look at the conditional probabilities: P(D or SD | M)=0.25, P(D or SD | A)=0.16, P(D or SD | E)=0.23, so the afternoon shift manager has the lowest rate of defects.

2. (a) c=1/15
   (b) E[X] = 11/3
   (c) V[X] = 14/9

3. (a) 8
   (b) 7
   (c) 2/sqrt(10)

4. $220

5. Expected payout = $1.5(10,000) – P(at least one snowstorm)(10,000) = $7,231.30

6. (a) $\binom{200}{16}$

   (b) $\binom{150}{16}/\binom{200}{16}$

   (c) $\sum_{k=4}^{16}\binom{50}{k}\binom{100}{16-k}$

7. Solve ¼ + p/2 = 190/300, so estimate p=0.767